

TETRACHORDAL ANALYSIS

Case Study: Arnold Schoenberg, *Sechs kleine Klavierstücke*, op.19 no.5

Already familiarized with the color-coded analysis, it is possible now to approach another set-class method based on tetrachordal analysis. This type of analysis explores concepts related to the techniques of segmentation (Straus 2005 and Hasty 1981), contour analysis (West 1987), similarity relationships (Quinn 2001 and Lord 1981) and fuzzy relationships (Quinn 1997).

An example showcasing the manner in which such segmentation is conducted and applied over the musical material is presented below, noting that the tetrachordal segmentation considers both the chordal as well as the melodic structures.

The image displays a musical score for Arnold Schoenberg's *Sechs kleine Klavierstücke*, op.19 no.5, annotated with tetrachordal analysis. The score is in 3/8 time and marked "Etwas rasch". The analysis is presented in three systems, each with a treble and bass clef staff. Red boxes highlight melodic tetrachords, while blue boxes highlight harmonic tetrachords. Fingerings and articulation marks are indicated above the notes. Labels such as "melodic tetrachord; omit C" and "melodic tetrachord; omit G#" are placed above specific melodic segments. The annotations show how the tetrachordal structure is applied to both the melodic lines and the harmonic accompaniment.

The analysis starts by assigning the proper PC (Pitch Class) to each pitch, making sure that the tetrachordal segmentation takes into

account all possible segmentations of the two types of structures. For an analytical example, we will discuss the fragment comprised of measures 5-11 and will first break down the tetrachordal structures.

Let us take measures 5-6 with pick-up for an example. First we identify a melodic contour comprised of the {Eb F# G F# A} pitches. Since F# is repeated, the repetition will be discounted and the resulting structure is the {Eb F# G A} melodic tetrachord. The chordal tetrachord {G Db F Bb} is also identified, this time as a vertical structure.

The tetrachordal analysis of measures 5-11 presents the sets and their most compact form by rotation on the serial dial:

Measures 5-6 (including the pick-up to measure 5)

The normal and prime forms for the melodic tetrachord {Eb, F#, G, F#, A} are as follows:

$$\{Eb, F\#, G, F\#, A\} \rightarrow \langle Eb F\# G A \rangle \rightarrow [3679] \rightarrow (0236)$$

The normal and prime forms for the harmonic tetrachord in measure 5 {G, Db, F, Bb} are as follows:

$$\{G, Db, F, Bb\} \rightarrow \langle Db F G Bb \rangle [157A] \rightarrow [57A1] \rightarrow (0258)$$

The melodic and the harmonic tetrachords identified so far belong to different set classes. However, they are linked via shared abstract subsets. First we reduce the trichordal subsets shared by these tetrachords to their prime form.

(0236)

$$[023] \rightarrow (013) \quad [026] \rightarrow (026) \quad [036] \rightarrow (036) \quad [236] \rightarrow (014)$$

(0258)

$$[025] \rightarrow (025) \quad [028] \rightarrow (028) \quad [058] \rightarrow (038) \quad [258] \rightarrow (036)$$

The prime forms suggest that these two tetrachords share a single trichordal subset (036). They also

Normal forms of the specific members of these two set-classes that are found in the melodic tetrachord and in the harmonic tetrachord are as follows:

$$\{Eb, G, A\} [036] \text{ \& \} \{G, Bb, Db\} [258]$$

Measures 7-8



The normal and prime forms for the melodic tetrachords {B, C, B, Eb, D} and {D, C#, D, B, C} are as follows:

{B, C, B, Eb, D} → <BCDEb> → [B023] → (0134)

{D, C#, D, B, C} → <BCC#D> → [B012] → (0123)

The normal and prime forms for the literal subset shared by these two tetrachords are [BCD] (013). Throughout these measures we observe a sustained chord {F#, G, A} whose normal and prime forms are {F#, G, A} → <F# G A> → [679] → (013).

The two melodic tetrachords and the sustained trichord are related by the shared (013).

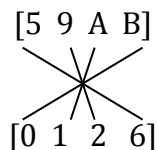
Measures 9-11 melody (including the pick-up to measure 9)

There are three melodic sets identified in measures 9-11:

- a. measure 9 with pick-up {A, A#, B, F} → <FAA#B> → [59AB] → (0126)
- b. measure 10 {F#, C, C#, D} → <CC#DF#> → [0126] → (0126)
- c. measures 10-11 {C, C#, D, Eb, D} → <CC#DEb> → [0123] → (0123)

The first two sets are inversionally-related and are also related via contour.

- a. The TnI relationship that links these sets:

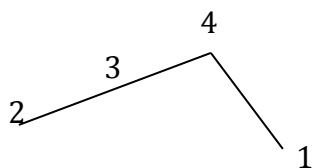


[0126] → T₀I [6AB0] → T₁₁ [59AB]

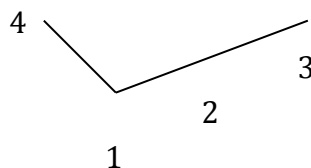
[59AB] → T₁₁I [0126]

- b. The CSEG for each set and the relationship between their CSEGs (identical or I-, R- or RI-related) reveals that they are RI-related.

{A, A#, B, F}



{F#, C, C#, D}



<2341>

| | 2 | 3 | 4 | 1 | INT4 |
|---|---|---|---|---|------------|
| 2 | 0 | + | + | - | INT3 |
| 3 | - | 0 | + | - | INT2 |
| 4 | - | - | 0 | - | INT1 |
| 1 | + | + | + | 0 | Main diag. |

<4123>

| | 4 | 1 | 2 | 3 | INT4 |
|---|---|---|---|---|------------|
| 4 | 0 | - | - | - | INT3 |
| 1 | + | 0 | + | + | INT2 |
| 2 | + | - | 0 | + | INT1 |
| 3 | + | - | - | 0 | Main diag. |

- c. There is an element of novelty in the way Schoenberg realized these PC sets in pitch space as well as the specific ordering of pitches he used.

From a compositional standpoint, working with intervals and pitches creates intrinsic similarities/identities between pitches and contour. If one set is simply transposed, the second set will have an identical contour. If one set is inverted, the contour will reflect that. The same stands for the RI operation. In certain cases, such as range displacement of a melodic line, the contour might not be identical,

however, in prime form will reflect the same similarity. In this case, Schoenberg chose to preserve the “obviousness” of the relationship between the two sets.

10. Regarding the set C identified in measures 9-11, a careful analysis shows that this set is related to the first two sets a and b by sharing a common trichord subset, namely (012):

a) Measure 9 with pick-up {A, A#, B, F} → <FAA#B> [59AB] → (0126)

b) Measure 10 {F#, C, C#, D} → <CC#DF#> [0126] → (0126)

c) Measures 10-11 {C, C#, D, Eb, D} → <CC#DEb> → [0123] → (0123)

Furthermore, this set relates to material in measures 7-8 by sharing the same (0123) with the second tetrachord as well as the diad [01] with both tetrachords.

Tetrachord1: {B, C, B, Eb, D} → <BCDEb> → [B023] → (0134)

Tetrachord2: {D, C#, D, B, C} → <BCC#D> → [B012] → (0123)

The framing pitches (highest, lowest, first and last pitches) for the melodic gesture in measure 9 (including the pick-up) form the trichord {A, B, F} and the framing pitches for the melodic gesture in measures 10-11 form the trichord {F#, C, D}.

{A, B, F} → <FAB> → [59B] → (026)

{F#, C, D} → <CDF#> → [026] → (026)

The transformation that links these trichords:

[026] → T_{0I} [6A0] → T₁₁ [59B]

[026] → T_{11I} [59B]

The melody of measures 9-11 (including the pick-up to measure 9) as a whole showcases two melodic tetrachords:

The normal and prime forms of its melodic framing pitches is comprised of {A, D, C, B} → <ABCD> → [9B02] → (0235).

The tetrachord (0235) is related with the sets (0123) and (0134) identified in measures 7-8 by three common tones [023] with the first one and two common tones [03] with the second one. All of them together share the two common tones [03].

Measures 9-11, harmony

The last beat of measures 9 and 10 showcases vertical tetrachords, namely:

- a. {Eb, G, C, B} → <BCEbG> → [B037] → (0148)
- b. {D, F#, C, F} → <CDFF#> → [0256] → (0146)
- c. {Bb, D, G#, C#} → <BbC#DG#> → <C#DGBb> → [127A] → (0169)
- d. {A, C#, G#, D} → <C#DG#A> → [G#AC#D] → [8912] → (0156)

The sets identified as b & c are members of the same set class. The two sets, (0146) and (0169) are subsets of the superset (01469). They also share the trichord (016), therefore they have three common tones. The first three chords (0148), (0146) and (0169) do not share any abstract trichordal set. However, the first two share (014) and the second and third sets share (016) this being also the abstract trichordal subset shared by the last three of these chords.

The chords in measures 9-11 could also be considered as vertical trichords, if excluding the melody line:

| | | | |
|-------|-------|-------|-------|
| [037] | [026] | [8T2] | [891] |
| (037) | (026) | (026) | (015) |

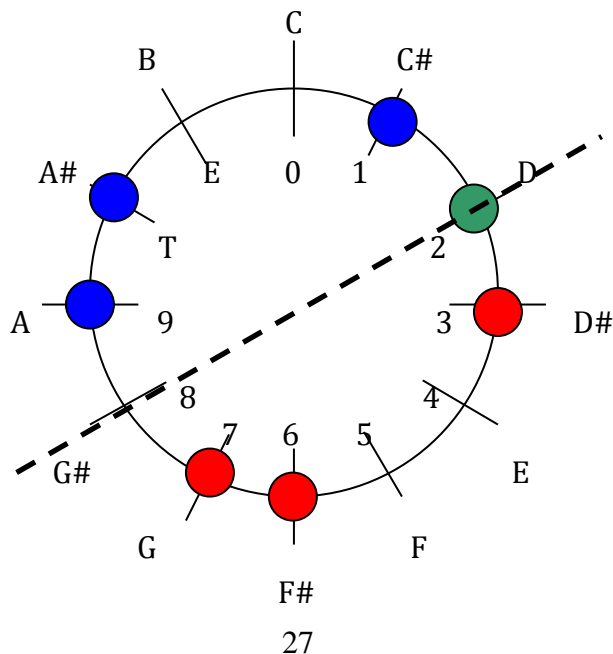
The *T_n (“fuzzy” T_n) which best describes the connection between the first 2 trichords [037] & [026] is expressed as [026]→*T₁[037]. The T_n (“crisp” T_n) which best describes the connection between the two (026) is expressed as [026]→T₈[8A2].

These transpositions relate to the intervallic connection among the sets identified in measures 9-11 as [026] could be considered an augmentation of the (016), which is meant to be the intervallic expansion which extends the musical language.

Yet another way to look at these harmonies is by focusing only on the bass-clef notes. In measure 9, the set consisting of the bass clef notes is {Eb, G, D, F#}→<D Eb F# G>→[2367]→(0145). In measure 10, the set consisting of the bass-clef notes is {Bb, D, A, C#}→<A Bb C# D>→[9A12]→(0145).

The sets identified in measures 9-10 bass clef display the TC (transpositional combinatorial) property. The two sets are identical symmetrical sets related by T₇. Therefore, their subsets will also be traspositionally related.

Each of these sets can be divided into transpositionally related subsets in two ways which are by division in trichords and diads or by symmetry axis. Furthermore, measures 9-10 reflect both conceptions of the sets’ TC potential as [2367]→T₇[9A12].

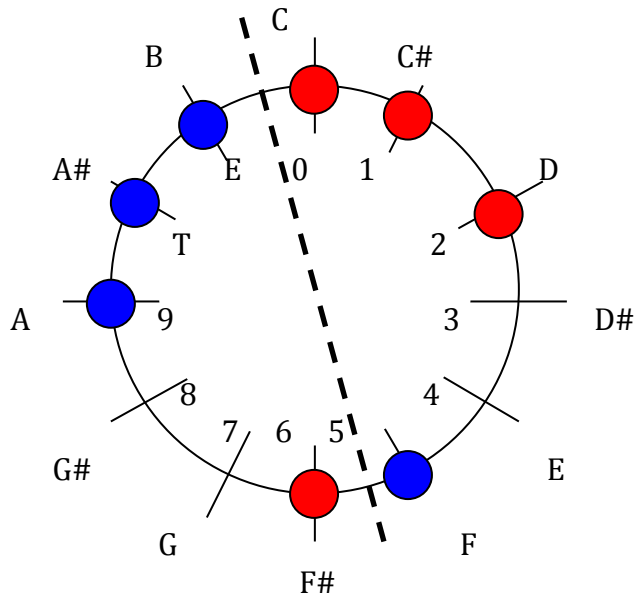


Some of the other tetrachordal set classes identified so far also have the TC property. Each TC set class identified divides into transpositionally related subsets.

Measure 9 with pick-up {A, A#, B, F} → <FAA#B> [59AB] → (0126)

Measure 10 {F#, C, C#, D} → <CC#DF#> [0126] → (0126)

[59AB] → T₁₁l [0126]



Some of the tetrachordal set-classes with TC potential identified above divide into trichordal subsets.

(0126)

[012] → (012) = may instances identified above

[016] → (016) = many instances identified above

[026] → (026) = many instances identified above

[126] → (015) = not identified above

Measure 5 and measure 9 (including their pick-ups)

These measures are related both melodically and harmonically.

The image displays two musical staves. The left staff, labeled 'Measure 5', features a melodic line with notes 3, 6, 7, 6, 9 and a harmonic tetrachord {Eb, F#, G, Bb} with fingerings 5, 1, 7. The right staff, labeled 'Measure 9', features a melodic line with notes 9, A, B, 5 and a harmonic tetrachord {Eb, G, C, B} with fingerings 0, 7. Labels include '(melodic tetrachord, omit C)' and 'harmonic tetrachord:'.

Measure 5 with pick-up showcases both a melodic tetrachord {Eb, F#, G, F#, A} → <EbF#GA> → [3679] → (0236) as well as a harmonic tetrachord {G, Db, F, Bb} → <Db F G Bb> [FGBbDb] → [57A1] → (0258).

Measure 9 with pick-up also showcases a melodic tetrachord {A, A#, B, F} → <FAA#B> [59AB] → (0126) and two harmonic tetrachords: {Eb, G, C, B} → <BCEbG> → [B037] → (0148) as well as {D, F#, C, F} → <CDF#F#> → [0256] → (0146).

Between the tetrachords in the two measures there are melodic connections as well as harmonic connections. As far as the melodic connections are concerned, we could consider (0236) an augmentation of (0126), based on three common tones, the [026] trichord being very present in the whole work.

In terms of harmonic connections, at a discrete level we could also consider the (025) a similar augmentation of the (014) trichord which is common to both (0148) and (0146) in measure 9. At a greater level, the (0148) tetrachord shares two common tones [0,8] with the tetrachord (0258) and it is a similar augmentation of the melodic/harmonic material.

As far as the discrete level of the subsets trichords are concerned, we have the following:

Measure 5

(0236)

[023]→(013) [026]→(026) [036]→(036) [236]→(014)

(0258)

[025]→(025) [028]→(028) [058]→(038) [258]→(036)

(0126)

[012]→(012) [016]→(016) [026]→(026) [126]→(015)

Measure 9

(0146)

[014]→(014) [016]→(016) [046]→(026) [146]→(025)

(0148)

[014]→(014) [018]→(018) [048]→(048) [148]→(037)

We see a reoccurrence of (014), (016) and (026) in both measures 5 and 9, in quite a symmetrical way: measure 5 (026) once and twice in measure 9 and vice versa for (014) while (016) is constant with one occurrence in both.

In measures 13-14, the four notes notated in the bass clef in the upper system form for the tetrachord {Ab, Bb, Db, Cb}→<Ab Bb Cb Db>→[8AB1]→(0235).

Another place in the analysis where a member of this set class appeared was in the framing pitches of the melody in measures 9-11 (including the pick-up to measure 9) as a whole.

(melodic tetrachord; omit C)

(melodic tetrachord; omit G#)

{A, D, C, B} → <ABCD> → [9B02] → (0235)

Measures 7-8

The sets found in the melodic lines of measures 7-8 relate to this set class:

{B, C, B, Eb, D} → <BCDEb> → [B023] → (0134)

{D, C#, D, B, C} → <BCC#D> → [B012] → (0123)

The tetrachord (0235) is related with the sets (0123) and (0134), identified in measures 7-8 by three common tones [023] with the first one and two common tones [03] with the second one. All of them together share the two common tones [03].

Measures 14-15, final two chords

The four trichords identified in measures 14-15 (normal and prime forms) are as follows:

- a. {D, Ab, C} → <CDAb> → [028] → (028)
- b. {B, F, A} → <FAB> → [59B] → [046] → (026)
- c. {D#, A, C#} → <C#D#A> → [913] → [035] → (025)
- d. {B, E, G#} → <EG#B> → [48B] → [046] → (026)

These trichords belong to two different set classes which were paired also in measures 9-10 in this movement.

Measures 9-10

In order to find out if the two hexachords formed by vertical pairs of trichords are members of the same set class, we need first to obtain their prime forms then compare them and their vectors.

{B, F, A, D, Ab, C} → <FAbABCD> → [589B02] → (023569)

| | | | | | |
|----|----|------|-----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| II | II | IIII | III | II | II |

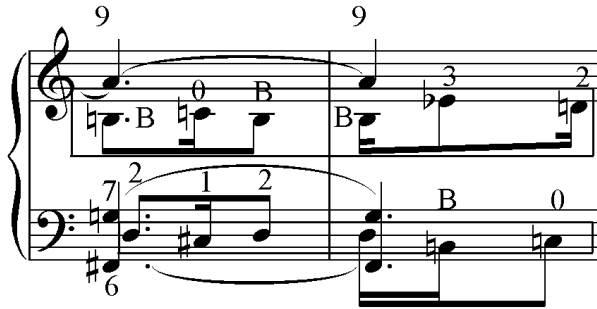
{B, E, G#, D#, A, C#} → <D#EG#ABC#> → <G#ABCD#E> → [89B034] → (013478)

| | | | | | |
|-----|---|-----|------|-----|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| III | I | III | IIII | III | I |

As a consequence, the two hexachords are not members of the same class as they don't have the same prime form nor the same vector.

A recurring issue in the preceding findings was the role played by trichords as common subsets linking tetrachords of different set-classes. Given that trichords seem to form the basis for most of the relationships asserted, there are a number of reasons to persist with tetrachordal analysis.

For the above mentioned trichords, the tetrachords are supersets that embed them and allow for modular expansion of the musical language in both, melody and harmony. Also, they expand the unit cell of structural development, allowing for new links between the subsets based on more than just common tones. Since the tetrachords themselves are in certain relationships, the subsets (micro level) can inherit those relationships and create a coherent structure at a macro level as well. Further observations regarding the role of common-tone connections in the passages considered thus far, may clarify further these links. Let us take for example the tetrachords in measures 7-8.



{B, C, B, Eb, D} → <BCDEb> → [B023] → (0134)
 {D, C#, D, B, C} → <BCC#D> → [B012] → (0123)

The common tones [BCD] → (013) create coherence in the melodic progression while at the same time allowing for new melodic elements such as [2] and [4] to be considered for further development, thus expanding the melodic material at a macro level and allowing for new possible connections to be developed in a modular fashion.

Several other passages from this work such as measures 1-3, measure 4 (including the pick-up) and measures 12-13 have yet to be examined, exploring how they relate to the passages analyzed thus far.

Measures 1-3

Etwas rasch (♩)

A number of secondary voice tetrachords are identified as follows:

{096A} → <69A0> → [69A0] → [0346] → (0236)

{1B25} → <125B> → [B12m5] → (0236)

{B257} → <257B> → [57B2] → (0268)

{2578} → <2578> → [2578] → [0356] → (0136)

{578A} → <578A> → [578A] → [0235] → (0235)

The frame notes of the melodic line which starts and ends on F, so the second F is disregarded:

Etwas rasch (♩)

Start: F / End: E / Highest: F / Lowest: D#

{F, E, F, D#} → <D#, E, F> → [345] → (012)

Segmentation of the melodic line:

{FAG#F#} → <FF#G#A> → [5689] → (0134)

{AG#F#D#} → <D#F#G#A> → [3689] → (0136)

{G#F#D#E} → <D#EF#G#> → [3468] → (0135)

{F#D#EF} → <D#EFF#> → [3456] → (0123)

Hexachord melodic line:

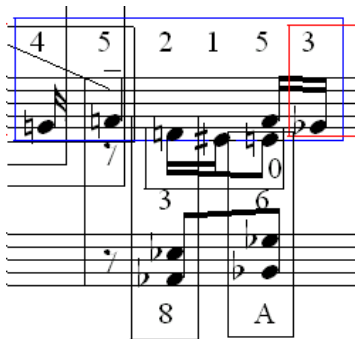
{F, A, G#, F#, D#, E, F} → <D#EFF#G#A> → [345689] → (012356)

Observations:

- The hexachord melodic line is a superset of the secondary voices tetrachords as well as the other ones shown above;
- Occurrence of (0136);
- (0123) is a superset of the melodic frame (012);
- Further segmentation in trichords to find inner connections is possible.

Measure 4 with auftakt (pick-up)

Measure 4 will be discussed in the context of the larger structural section of measures 4-8.



Since [Eb] at the end of measure 4 is segmented with measure 5, we will segment {EFDC#F} using the pick-up (auftakt) for measure 4. The following segmentations make sense due to the melodic line and the accompaniment harmony:

Melodic tetrachord: {EFDC#F} → <C#DEF> → [1245] → (0134)

Note: The (0134) set appears in the melodic line of the measures 1-3, 7-8, 9-11 and 14-15 underlining its importance for the entire architecture of the piece.

Melodic trichord:

{D C# C} → <CC#D> → [012] → (012)

Note: The chromatic trichord (012) appears also very often as a subset of larger sets such as in measures 5, 7-8, 9-11, and so on.

Harmonic Trichords:

{Ab Eb D} → <D Eb Ab> → [238] → (016)

{Bb Gb C} → <C Gb Bb> → [6A0] → (024)

Measures 12-13

Measures 12-13 will be analyzed in the context of the larger closing section of mm. 12-15. The following segmentations make sense due to the melodic line and the accompaniment harmony:

The image shows a musical score for measures 12 and 13. The score is written for piano, with a treble and bass clef. Measure 12 starts with a treble clef and a bass clef. The treble clef has a melodic line with notes G4, F#4, E4, D4, and C4. The bass clef has a harmonic accompaniment with notes G2, B2, and D3. Red boxes highlight the melodic line in measure 12 and the harmonic accompaniment in measure 12. Blue boxes highlight the melodic line in measure 13 and the harmonic accompaniment in measure 13. The score is annotated with numbers 1 through 9, indicating specific notes or intervals.

Melodic-harmonic tetrachord:

{AFEbD} → <2359> → [2359] → (0137)

{GbBbGB} → <GbGBbB> → [67AB] → (0145)

{A#C#G#B} → <G#A#BC#> → [8AB1] → (0235)

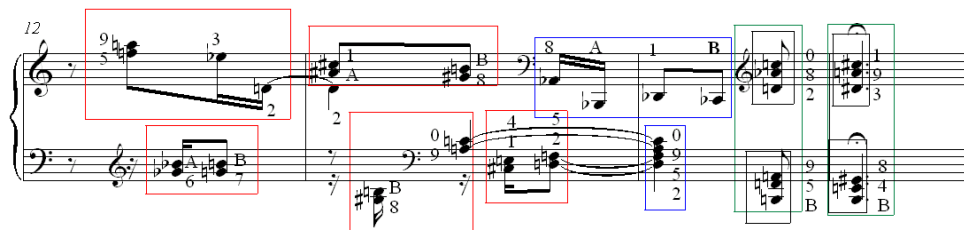
{G#BAC} → <G#ABC> → [89B0] → (0134)

{C#EDF} → <C#DEF> → [1245] → (0134)

Note: it seems again that (0134) plays an important role in this work since it appears again in measures 4 and 12-13.

Measures 12-15

The full segmentation of measures 12-15 is presented below:



Other segmentations such as: only highest or lowest pitches; only melodic pitches; only harmonic pitches; framing pitches, and so on, are possible and may reveal connections at a deeper, more abstract level. Moreover, a detailed study of all the subset trichords can also show more abstract levels of connections between the segmented fragments.